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Vedic Origin of Zero

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Foreword

The invention of zero has proved a game changer in the human history of science and technology. No scientific advancement would have been possible without the invention of zero. In fact, its invention revolutionized everything in science and technology. The famous mathematician G.B. Halsted and Pierre Simon De Laplace, a noted French mathematician and astronomer during the eighteenth century give credit for the invention of zero to Indians.

Halsted (1912:20) observes thus:

"The importance of the creation of zero mark can never be exaggerated. This giving to airy nothing, not merely a local habitation and a name, a picture, a symbol, but helpful power, is the characteristic of the Hindu race whence it sprang. It is like coining Nirvana into dynamos. No single mathematical creation has been more potent for the general on-go of the intelligence and power"

Laplace (Ifrah, 2000) observes:

"The ingenious method of expressing all numbers by means of ten symbols, (each symbol having a value of position as well as an absolute value) emerged in India. The idea seems so simple nowadays that its significance and profound importance is no longer appreciated. Its simplicity lies in the way it facilitated calculations and placed arithmetic foremost among useful inventions. The importance of this invention is more readily appreciated when one considers that it was beyond the two greatest men of antiquity, Archimedes and Apollonius."

Although the presence of zero has been traced to many ancient civilizations including India, the question of its origin is still a moot point among historians and mathematicians. As such, in the present monograph, a humble attempt will be made to gather data from the Vedic and Post Vedic Sanskrit literature to understand the origin of zero; the philosophy behind its origin; various denominations of zero used in various phases of Indian civilization; and the philosophy and epistemology behind the symbolic representation of zero (0) in India. Here I should not forget to acknowledge Alois Heinrich who went through the proofs of this monograph and designed the cover page.

Prof. Ravi Prakash Arya

Chicago, USA

Introduction

The Vedas are the oldest ever literature in the human history of the globe. As per ancient Indian tradition, all branches of knowledge have their roots in the Vedas.

वेदोऽखिलो धर्ममूलम्।

vedo'khilodharmamūlam

[Meaning] Veda is the source of all knowledge-metaphysical, astrophysical and physical, morality and ethics.

The entire later Vedic and post-Vedic literature speaks highly of the Vedic seers and the Vedas for their contribution to science and civilization on the globe. As such when we search for the origin and the development of various sciences in the Indian context from the literary point of view, the Vedas are screened first to find out the seed germinated in them. As in the case of other schools of thought and sciences, the science of mathematics too in India goes back to the Vedic period for its origin and development.

Algebra, Arithmetic including Geometry had their origin in India.

According to Sarda (2007: 295), "In mental abstraction and concentration of thought, Indians are proverbially happy. Apart from direct testimony on the point, the literature of the Hindus furnishes unmistakable evidence to prove that the ancient Hindus possessed astonishing powers of memory and concentration of thought. Hence all such sciences and branches of study as demand concentration of thought and a highly-developed power of abstraction of the mind were highly cultivated by the Indians. The science of mathematics, the most abstract of all sciences, must have had an irresistible fascination for the minds of the Indians. Nor are there proofs wanting to support this statement. The most extensive cultivation that astronomy received at the hands of the Indians is in itself proof of their high proficiency in mathematics. The high antiquity of Hindu astronomy is an argument in support of the still greater antiquity of their mathematics. That the Indians were selected by nature to excel all other nations in mathematics, is proved by her revealing to them the foundation of all mathematics. It has been admitted by all competent authorities that the Indians were the inventors of the numerals".

The great German critic, Schlegel (Sarda, 2007:295) says that the Hindus invented "the decimal ciphers, the honour of which, next to letters the most important of human discoveries, has, with the common consent of historical authorities, been ascribed to the Hindus."

Prof. Macdonell (1900:424) says, "In science, too, the debt of Europe to India has been considerable. There is in the first place, the great fact that the Indians invented the numerical figures used all over The world. The influence that the decimal system of reckoning dependent on those figures has had not only on mathematics but on the progress of civilization in general, can hardly be overestimated. During the eighth and ninth centuries, the Indians became the teachers in arithmetic and algebra of the Arabs, and through them of the nations of the west. Thus, though we call the latter science by an Arabic name, it is a gift we owe to India."

Sir M. Monier Williams (1875:124) says, "From them (Indians) the Arabs received not only their first conceptions of algebraic analysis, but also those numerical symbols and decimal notations, now current everywhere in Europe, and which have rendered untold service to the progress of arithmetical science." Says Manning (1869, Vol. 1: 376), 'To whatever cyclopedia, journal or essay we refer, we uniformly find our numerals traced to India and the Arabs recognized as the medium through which they were introduced into Europe."

W. W. Hunter (1881:219) also says, "To them (the Indians) we owe the invention of the numerical symbols on the decimal scale. The Indian figures 1 to 9 are abbreviated forms of initial letters of the numerals themselves, and the zero, or 0, represents the first letter of the Sanskrit word for śūnya (श). The Arabs borrowed them from the Hindus and transmitted them to Europe."

Professor Weber (1878:256) says, "It is to them (the Hindus) also that we owe the ingenious invention the numerical symbols, which in like manner passed from them to the Arabs, and from these again to European scholars. By these latter, who were the disciples of Arabs, frequent allusion is made to the Indians uniformly in terms of high esteem; and one Sanskrit word even (uccha) has passed into the Latin translations of Arabian astronomers."

Mrs. Manning (1869, Vol.1 : 374) says, "Compared with other nations, the Hindus were peculiarly strong in all the branches of arithmetic."

Professor Weber (Sarda, 298), after declaring that the Arabs were disciples of the Hindus, says, "The same thing (i.e. the Arabs borrowing from Hindus) took place also in regard to algebra and arithmetic in particular, in both of which it appears the Hindus attained, quite independently, to a high degree of proficiency."

W.W. Hunter (1881:219) also says that the Hindus attained a very high proficiency in arithmetic and algebra independently of any foreign influence.

The English mathematician, Prof. Wallace (Edinburgh Review, 1818: 147), says, "The Līlāvati treats of arithmetic, contains not only the common rules of that science, but the application of these various questions of interest, barter, mixture, combinations, permutations, sums of progression indeterminate problems, and mensuration of surface and solids. The rules are found nearly as simple as in the present state of the analytical investigation. The numerical results are readily deduced, and if they are compared with the earliest specimens of Greek calculation, the advantages of the decimal notation are placed in a striking, light." It may, however, be mentioned that Līlāvati, of which Professor Wallace speaks, is a comparatively modern manual of arithmetic; and to judge the merits of Indian arithmetic from this book is to judge the merits of English arithmetic from Chamber's manual of arithmetic.

It may be added that the enormous extent to which numerical calculation goes in India, and the possession by the Indians of by far the largest fable of calculation, are in themselves proofs of the superior cultivation of the science of arithmetic by the Indians and their invention of śūny (zero). In fact, the invention of śūny (zero) was the basis of the invention of decimal system and place value notations by the Indians. That is why they used the numbers up to the endless limit. Even the *Ṛgveda* describes the name of 7 notational places from eka (units) to prayuta (millions) and the *Yajurveda* (17.2) mentions name of 12 notational places from eka (units) to parārdha (billions).

Due to a lack of knowledge of zero, decimal system, and place value notations, Arabs, Greeks, and Romans could not use big numbers.

The Greeks used myriad (10^4) as the limit of counting while Romans defined mille (10^3) as the largest number. The Indians carried the decimal numeration, naming the successive power of a number (usually 10), far beyond any other civilization of the past.

Having seen this Al-Biruni (Sachau, 1964, Vol.1: 174) got puzzled and criticized Bharatiyas for their passion for large numbers. He observes:

"I have studied the names of the orders of the numbers in various languages with all kinds of people with whom I have been in contact, and have found that no nation goes beyond a thousand. The Arabs, too stop with thousand, which is certainly the most correct and the most natural thing to do. Those, however, who go beyond the thousand in their numeral system are the Hindus, at least in their arithmetical technical terms".

The above reference of Al-Biruni (943-1048 C.E.) proves beyond an iota of doubt that by his time, people of the rest of the world, including Arabs, were not aware of zero or its placement value. That is why all numbers like ten, hundred, and thousand were denoted by separate symbols. It were Indians who knew the zero and its placement value. That is why they were able to count in billions and trillions which surprised and baffled Al-Birunī.

I am sure if Al-Birunī were to write his books today, he would not have criticized the Indians for their fondness for large numbers. Al-Birunī mentioned 10^{19} as the largest number used by the Hindus. The Babylonians did not use numbers beyond one thousand. However, for some reason, the Arabs later on restricted themselves to

100 only.

During the first century B.C., in Lalitavistara, a book on the life of Lord Buddha, in a dialogue between Lord Buddha and Arjuna, Tallakṣaṇa (10^{53}) is defined. As the story goes when Gautama reached manhood, he courted Gopa, the daughter of Kind Daṇḍāpaṇīin the tradition of svayaṁvara a tradition where the young men demonstrated their abilities in the presence of the bride and her family. The bride, after consultation with the family, identifies the man of her choice of marriage. Gautama demonstrated public proof of his scholarly abilities by debating (śāstrārtha) with Arjuna, the state mathematician of King Daṇḍāpaṇī. Arjuna asked Buddha to count numbers. Buddha defined Koṭi (10^7) and in steps of 100 defined numbers all the way up to Tallakṣaṇa (10^{53}).

The place value system is now commonly used throughout the civilized world. Without the zero and the place-value, the Indian numerals would have been no better than many other civilizations, and would not have been adopted by all the civilized peoples of the world.

Ease of Calculation: The invention of the zero and place value system also provided ease of calculation in the Indian system. In the Indian system, at the first glance, you may recognize greater and smaller numbers according to their digits. For example, 1892 is greater than 993, because 1892 has four digits and 993 three digits.

So far as the Roman system is concerned, 100 will be written as 'C' and 99 will be written as 'IC', smaller number has greater digits. Similarly, 18 will be written as XVIII and 90 will be written as XC.

Ignorance of zero: The above observation of Al-Biruni clearly shows that Arabs and the rest of the world were oblivious of the concept of zero and the place value notations till the middle of the 10th century A.D.

According to Mr. Clark (1929:217), Arabic literary tradition, as generally interpreted, declares that the numerals with zero and place value were invented by the Indians, and they were adopted by the Arabs during the last quarter of the eighth century A.D.

Indian zero was introduced by Persian scholar Abu Jafar Muh Al-Khwarizmi (780-850 CE) in the 9th century. He wrote a book in Arabic which was translated into Latin under the title '*De Numero Indorum*' (The Hindu Art of Reckoning). The manuscript of this book is Later copied in the 13th century and held by Cambridge University Library vide Ms. Ii.vi.5.. It was translated into English by John N. Crossley and Alan S. Henry in 1990 which was published in *Historia Mathematica* 17 (1990), 103-131. The following excerpt from the same translation confirms the above fact.

Algorizmi [Al-Khwarizmi] said: ... "I had seen that the Indians had set up 9 symbols in their universal system of numbering ... for the sake of its ease and brevity, so that this work, to be sure, might be made easier for the seeker after arithmetic. ... So they made 9 symbols,whose forms are these: 9 8 7 6 5 4 3 2 1 ...

But when X was put in the place of one and was made in the second place, and its form was the form of one, they needed a form for the tens because of the fact that it was similar to the form of one, so that they might know by means of it that it was 10. So they put one space in front of it and put in it a little circle like the letter 'o', so that by means of this they might know that the place of the units was empty and that no number was in it except the little circle, which we have said occupied it, and it is shown that the number that is in the following place was a ten and that this was the second space, which is the place of the tens. And they put after the circle in the aforesaid second place whatever they wished from the number of tens from whatis between X and XC and these are the forms of the tens: the form of X is thus (10), the form of XX [is] 20. And likewise, the form of XXX is thus (30), [Fol. 105r] and so on up to IX (sc. tens), there will be, clearly, a circle in the first place and a character pertaining to the number itself in the second place. Moreover, one must know this, that the character that signifies one in the first place, in the second signifies 'X' (10), in the third 'C' (100) and in the fourth Ī (1000). And likewise, the character that in the first place signifies two, in the second signifies XX (20) and in the third CC (200) and in the fourth ĪĪ (2000) and understand likewise about the rest."

The above statement of Al-Khwarizmi, proves two things without any shadow of a doubt.

1. Arab and the Western world was unaware of zero until the time Al-Khwarizmi brought Indian zero into thelimelight.

2. The Indians were using place value notations from the very beginning which is why long numbers could be written.

The most glaring fact is that the western world remained oblivious of the concept of zero till 1582 A.D., until the time of Pope Gregory who amended the Julian Calendar into the Gregorian Calendar. This was the reason why the Europeans used to calculate 1 B.C. before 1 A.D. Had they been aware of zero, they would have not calculated 1 AD after 1 BC.

Now we would make a humble attempt to find out the origin, nomenclature, and symbol of zero in Vedic, later Vedic and post Vedic texts.

From the study of Vedic texts, one is able to glean the fact that both place value notation and zero were in ordinary use in the Vedic literature. The place value system, decimal system, and powers of ten attested in the Vedic, later Vedic, and post-Vedic texts are by no means the glaring proofs of the invention of zero by Indians in the hoary past. Now we shall take stock of the status of zero in respect of the Vedic and post-Vedic Sanskrit literature.

Powers of Ten

The concept of powers of 10 cannot be invented without the knowledge of zero. We all know the power of 10 is the number 1 followed by n zeros, where n is the exponent and is greater than 0; for example, 10^6 is written as 1,000,000. When n is less than 0, the power of 10 is the number 1 n places after the decimal point; for example, 10^{-2} is written as 0.01. When n is equal to 0, the power of 10 is 1; that is, $10^0 = 1$.

The contents of power of ten in the enumeration are the Vedic invention. We find the origin of this concept in the Ṛgeveda itself. Here we shall quote a few mantras supporting the above hypothesis. A mantra in the *Ṛgveda*(8.1.5) says:

ये ते सन्ति दशग्विनः शतिनो ये सहस्रिणः ।
अश्वासो ये ते वृष्णा रघुद्रुवस्तेभिर्नस्तूयमा गहि ॥

ye tesantidaśagvinaḥśatino ye sahasriṇaḥ;
aśvāso ye tevṛṣṇāraghudruvastebhirnastūyamāgahi.

[Meaning] Come quickly with those thy horses which are vigorous and fleet, and which are traverses of tens (10^1), or hundreds (10^2) or thousands (10^3) of leagues.

This concept is further eleborated in the *Yajurveda*. The *Yajurveda* (**(17.2)** illustrates how the power of eka (one) increases by 10 times when zero is suffixed to them.

इमा मेऽअग्नऽइष्टका धेनवः सन्त्वेका च दश च दश च शतं च शतं च सहस्र च सहस्र चायुतं चायुतं च नियुतं च नियुतं च प्रयुतं चार्बुदं च न्यर्बुदं च समुद्रश्च मध्यं चान्तश्च परार्धश्चैता मे ऽअग्न ऽइष्टका धेनवः सन्त्वमूत्रामूष्मिँल्लोके ।
imāme'agna'iṣṭakādhenavaḥsantvekā cha daśa cha daśa cha śataṁ cha śataṁ cha sahasra cha

sahasrachāyutaṁchāyutaṁ cha niyutaṁ cha niyutaṁ cha prayutaṁchārbudaṁ cha nyarbudaṁ cha samudraśchamadhyaṁchāntaśchaparārdhaddachaitā me 'agna 'iṣṭakādhenavaḥsantvamūtrāmūṣmiṁ̊lloke.

[Meaning] "O Agni may these iṣṭakā (bricks) of yajña altar provide me desirous fruits like a milch-kine. They increase by 10 times like numerals when suffixed with zero; e.g. 10^0 (one) becomes 10^1 (ten); 10^1 becomes 10^2 (100); 10^2 (100) becomes 10^3 (1,000); 10^3 (one thousand) becomes 10^4 (10,000 or ayuta); 10^4 (10,000 or ayuta) becomes 10^5 (niyuta or 100,000); 10^5 (100,000 orniyuta) becomes 10^6 (1,000,000 or prayuta); 10^6 (a Prayuta or one million) becomes an 10^7 (arbuda or ten million); 10^7 (arbuda) becomes 10^8 (nyarbuda or one hundred million), 10^8 (a nyarbuda becomes 10^9 (a Samudra or one thousand millions); 10^9 (a samudra becomes 10^{10} (madhya or a ten thousand millions); 10^{10} (a madhya becomes 10^{11} (anta or a hundred thousand millions); 10^{11} (an anta) becomes 10^{12} (parārdha or a million million or a billion). May these bricks act as milch-kine in providing fruits in yonder world and in this world.

In the above mantra of the *Yajurveda*, powers of 10 have been counted starting from 10^0 to 10^{12}. It is revealed here that zero increases the magnitude of numerals by 10 times when suffixed to them. Inversely it also points out a place or positional value system, when the place orposition of a number increases from right to left, its magnitude is increased by 10 times. Not only this the place names have also been given from eka (unit) to parārdha (billion).

The above list of numerals is also reproduced in the *Taittirīya Saṁhitā* (4.4.11), the *Kāṭhaka Saṁhitā* (17.10.31), and the *Maitrāyaṇī Saṁhitā* (2.18.14).

Āryabhaṭṭa restated the same fact when he claims it to be ancient knowledge. According to him (Āryabhaṭīyam, GaṇitaPāda, 2): if the position of a number increases from right to left, its magnitude increases by 10 times.

एकं च दश च शतं च सहस्रं त्वयुतनियुते तथा प्रयुतम्।

कोट्यर्बुदं च वृन्दं स्थानात्स्थानं दशगुणं स्यात्।।

ēkaṁ cha daśa cha śataṁ cha

sahasraṁtvayutaniyutetathāprayutam;

koṭyarbudaṁ cha vṛndaṁ sthānātsthānaṁdaśaguṇaṁsyāt.

[Meaning] 10^0 (eka/ 1),

10^1 (daśa/10),

10^2 (śatam/100),

10^3 (sahasram/1,000),

10^4 (ayutam/10,000),

10^5 (niyutam/100,000),

10^6 (prayuta/1,000,000),

10^7 (koṭi/10,000,000),

10^8 (arbudam/100,000,000) and

10^9 (vṛnda/1,000,000,000) are, respectively, each ten times the preceding, because of their positional/place value or notational places. With the increase in the position of number 1 from right to left, its magnitude is increased by 10 times. Inversely, zero increases the magnitude of numbers by 10 times when suffixed to them.

Here it may be noted that Āryabhaṭa (Golādhyāya, 50) does not tell this his invention, but he says that he owes this all knowledge to the Brahma-siddhānta which was in vogue in the hoary past well before his period.

आर्यभटीयं नाम्ना पूर्वं स्वायम्भुवं सदा नित्यम्।
सुकृतायुषोः प्रणाशं कुरुते प्रतिकंचुकं योऽस्य।।

āryabhaṭīyaṁnāmnāpūrvaṁsvāyambhuvaṁsadānityam;
sukṛtāyuṣoḥpraṇāśaṁkurutepratikaṁchukaṁyo'sya.

[Meaning] This '*Āryabhaṭīya*' is the same work as the ancient Brāhma-siddhānta (which was revealed by Svāyambhū) and as such, it is true for all times. One who finds fault with it shall lose his fame and longevity.

The Place-Value Notations

The place value notation as pointed out above was indicated with the help of zero. In the present days also in elementary schools in India, the student is taught the names of several notational places and is made to denote them by zeros arranged in a line. These zeros are written as 000000000

The teacher points out the first zero on the right and says 'units', then he proceeds to the next zero saying 'tens' and so on. The student repeats the names after the teacher. This practice of denoting the notational places by zeros can be traced back to the time of the Vedas.

In the Vedas, we come across the names of various notational places like eka, daśa, sahasra, ayuta, etc. up to parārdha.

In the *Ṛgveda,* we find names of the notational places from eka (units) to Prayuta (millions). Examples are:

Eka (units): Ṛgveda, 3.29.15; 10.59.9; 1.20.7; 5.2.17

Daśa (tens): Ṛgveda, 1.53.6; 8.1.5; 10.122.13

Śata (hundreds): Ṛgveda, 1.24.9; 4.26.3; 8.32.18

Sahasra (thousands): Ṛgveda, 1.80.9; 4.26.3; 8.2.41; 8.32.18

Ayuta (ten thousands): Ṛgveda, 4.26.4

Niyuta (hundred thousands): Ṛgveda, 1.134.2

Prayuta (millions): Ṛgveda, 1.51.6

In the *Yajurveda* (17.2), as already explained above, we find names of 12 notational places from eka (unit) to parārdha (billion).

The uniqueness of the Vedic system lies in the fact that the position of a number qualifies its magnitude.

Tens, hundreds, or thousands were not represented by different signs like Egyptians, Chinese, Romans, and other civilizations. They are represented by using digits in different positions. Notice that the one in the second place is 10 (ten), in the third place is 100 (hundred), and in the fourth place is 1,000 (thousand).

For example, the *Ṛgveda* (10.52.6) following the place value notation mentions "three thousand and three hundred and thirty-nine (3339)" as the count of people in a yajña.

त्रीणि शता त्री सहस्राण्यग्निं त्रिंशच्च देवा नव चासपर्यन्।

औक्षन्घृतैरस्तृणन्बर्हिरस्मा आदिद्धोतारं न्यसादयन्त।।

trīṇiśatātrīsahasrāṇyagniṁtriṁśachchadevānavachāsapa ryan;
aukṣanghṛtairastṛṇanbarhirasmāādidhotāraṁnyasādaya nta.

[Meaning] Three thousand (3000), three hundred (300), thirty (30) and nine (9) scholars performed yajña, they gave oblations of ghee, they sit on the sacred grass strewn around the altars.

In a similar manner, the *Atharvaveda* (8.8.7) mentions a hundred (100), thousand (1000), ten thousand (10,000), and hundred-thousand (1000,000) as under:

बृहत्ते जालं बृहत इन्द्र शूर सहस्रार्घस्य शतवीर्यस्य ।

तेन शतं सहस्रमयुतं न्यर्बुदं जघान शक्रो दस्यूनामभिधाय सेनया ॥

bṛhattejālaṁbṛhataindraśūrasahasrārghasyaśatavīryasya ;
tenaśataṁsahasramayutaṁnyarbudaṁjaghānaśakrodasy ūnāmabhidhāyasenayā.

[Meaning] O Indra (lightning in clouds) your network is great, for your qualities, you are appreciated by thousands of scholars. With your help, hundreds of works can be performed. With his power, Indra (lightning in clouds) has discharged a hundred, thousand, ten thousand, and hundred thousand clouds.

The evidence of the use of the word 'notational place' is also furnished by the *Anujogadvāra-sūtra* (100 B.C.) in the sūtra 142 (Datta, 1912:12). At one place it is stated, "a number which when expressed in terms of the denominations, koṭi-koṭi, etc., occupies twenty-nine places (sthānas), or it is beyond the 24th place and within the 32nd place, or it is a number obtained by multiplying the sixth square (of two) by (its) fifth square, (i.e., 2^{96}), or it is a number which can be divided (by two) ninety-six times." Another big number that occurs in the Jaina works is the number representing the period of time known as ŚīrṣaPrahelikā. According to the commentator Hemachandra (1st century A.D.), this number is so large as to occupy 194 notational places (aṅka-sthānāni). It is also stated to be $(8,400,000)^{28}$.

The first foreign proof of the use of place value notation in India was given by Al-Khwarizmi (9th century), a Persian scholar, already discussed above.

Left to Right Notation

Here it is important to know that in a positional or place-value system, a number, represented as x_4, x_3, x_2, x_1 can be constructed as follows:

$x_1 + (x_2 \times 10^1) + (x_3 \times 10^2) + (x_4 \times 10^3)$

Where x_1, x_2, x_3, and x_4 are non-negative integers that have magnitudes less than the chosen base (ten in our case). As you may notice, the magnitude of a number increases from right to left. For example, the number 1961 will be written as

$1 + (6 \times 10^1) + (9 \times 10^2) + (1 \times 10^3)$

The above notation is strictly followed in the writing in Sanskrit. The numbers were written in the Sanskrit language from left to right, but numerically they were placed from right to left, as per Vedic rule described in the *Samayochita Padma Ratna Malika* (śloka No. 48)

अंकानां वामतो गतिः।

aṁkānāṁvāmatogatiḥ

[Meaning] Numerically the numbers are placed from left to right.

For example, the following mantra of the *Atharvaveda* (8.2.21) follows the left-to-right notation while describing the number of years in a Kalpa period.

शतं तेऽयुतं हायनान् द्वे युगे त्रीणि चत्वारि कृण्मः।
इन्द्राग्नी विश्वेदेवास्तेऽनुमन्यन्तामहृणीयमानाः।।

śataṁte'yutaṁhāyanāndveyugetrīṇichatvārikṛṇmaḥ;
indrāgnīviśvedevāste'numanyantāmahṛṇīyamānāḥ.

[Meaning] Viśvedevāḥ (The Galaxy) takes 4,32,00,00,000 (4.32 billion) years to complete one revolution. Here *Visvedevāḥ* is representative of Galaxy."

The above mantra states to write 2,3 and 4 after 100 Ayutas. One Ayuta is equal to 100,000. As such 100 Ayutas will be equal to one million (seven zeros. When 2, 3, and 4 are placed from left to right before a million we get the number 4, 320,000,000 which is the period of a Kalpa or say the time of our Galaxy's revolution around the Supergalactic centre (SvāyambhūvaMaṇḍala).

Later Vedic and Post-Vedic Sanskrit literature is full of such examples.

Word Numerals and Place Value Notation

In addition to the above, it is pertinent to note here that a system of expressing numbers by means of words arranged as in the place-value notation was developed and perfected in India in the earliest phases of her ancient history. In this system, the numerals are expressed by names of things, beings, or concepts, which naturally or in accordance with the teaching of the Śāstras, connote numbers. Thus the number one may be denoted by anything that is markedly unique, e.g. the moon, the earth, etc.; number two by the eyes, the hands, etc; and similarly others. The zero was denoted by words meaning ख (kha),i.e. void, आकाश (ākāśa), i.e. space, ब्रह्म (Brahman), पूर्ण (pūrṇa), i.e. full or absolute, शून्य (śūnya), i.e. empty.

This system was used in works on astronomy, mathematics, and metrics, as well as in the dates of inscriptions and manuscripts. The ancient Indian mathematicians and astronomers wrote their works in verse. Consequently, they strongly felt the need for a convenient method of expressing the large numbers that occur so often in astronomical works and the statement of problems in mathematics. The word numerals were invented to fulfill this need and soon became very popular. They are used even up to the present day, whenever big numbers have to be expressed in Sanskrit verse.

Bhattotpala (Dvivedi, 1996: 163) in his commentary on the *Bṛhat-saṁhitā* has given a quotation from the original *Pauliśa-siddhānta* in which the word numerals are used. The number expressed in this quotation is

ख (0) ख (0) अष्ट (8) मुनि (7) राम(3) अश्विन् (2) नेत्र (2) अष्ट

(8) शर (5) रात्रि /चन्द्रमा 1)

The above quotation will be numerically represented as 1,582,237, 800.

Sanskrit literature is full of such examples, for the brevity of space and time, only one example has been quoted.

The words denoting the numbers from one to nine and zero, with the use of the principle of place-value, give us a very convenient method of expressing numbers by word chronograms. To take a concrete case, the number 1230 may be expressed in many ways:

1. kha-guṇa-kar-ādi,
2. kha-loka-karṇa-candra,
3. ākāśa-kāla-netra-dharā

It will be observed that the same number can be expressed in hundreds of ways by word chronograms. This property makes the word numerals especially suitable for inclusion in metres. To secure still greater variety, the numbers beyond ten are also sometimes denoted by works.

We also find an ancient record of the place-value notation coming from Vāsumitra, a leading figure of Kaniṣka's Great Council. According to Hiuen Tsang (602-664), Kuṣāṇa King Kaniṣka (144-178 A.D.) called a convocation of scholars to write a book, Mahāvibhāṣa. Four main scholars under the chief monk, named Pārśva, wrote the book in 12 years. Vāsumitra was one of the four scholars. In this book, Vāsumitra tried to explain that matter is continually changing as it is defined by an instant (time), shape, mass, etc. As time is continually changing, therefore, the matter is different in each situation although its appearance and mass do not change. He used an analogy of place-value notation to emphasize his point. Just as the location of digit one (1) in the place of hundred is called hundred (100) and in the place of thousand (1,000) is called thousand, similarly matter changes its state (avasthā) in different time designations. New explanations are generally given in terms of known and established facts. Thus the very reason Vāsumitra used place-value notation as an example established that the place-value notation was considered an established knowledge during the early Christian era. (Kumar, 2014; 67)

Base Ten Decimal System

Here it is important to know that the base ten (Decimal) numeral system is also Vedic in origin. In fact, the words 'deca', 'deci', 'decimal', 'ten' etc are the offshoots of the Sanskrit word 'Daśa' (दश).

The decimal system represents numbers as linear combinations of the powers of ten as shown above, e.g.

$10 = 10^1$

$20 = 2 \times 10^1$

$30 = 3 \times 10^1$

$16 = 6 + 10^1$

This is possible when the knowledge of zero is present. Several other civilizations such as the Egyptians, Chinese and Romans did not invent zero, so they used an additive decimal system. In their system, multiple repetitions of symbols were used and added to increase the magnitude. For example, in the Roman system x means 10 and xx (10+10) became 20, xxx (10+10+10) became 30, xvi (10+5+1) became 16. So for want of the invention of zero they have to repeat the symbols again and again.

It is India and only India that gave the world an ingenious method of expressing all numbers by means of 10 symbols, each symbol receiving a value of the position and as well as an absolute value.

We receive first indication of decimal system from the *Yajurveda* (27.33):

एकया-दशभिः (*ēkayā-daśabhiḥ*) 1 x 10

द्वाभ्यां-विंशति (*dvābhyāṁ-viṁśati*) 2 x 10

तिसृभिः त्रिशंता (*tisṛbhiḥtriśaṁtā*) 3 x 10

The *Atharvaveda* (5.15.1-11) forms numbers from 1 to 1000 on the basis of decimal system

एका च मे दश च मेऽपवक्तार ओषधे ।

ऋतजात ऋतावरि मधु मे मधुला करः ॥१॥

ēkā cha me daśa cha me'pavaktāraōṣadhe;
ṛtajātaṛtāvarimadhu me madhulākaraḥ.

[Meaning] O herb used in performing yajña and full of juicy potentialities make us regain health if we are attacked by one disease and (1×10^1) ten.

द्वे च मे विंशतिश्च मेऽपवक्तार ओषधे ।

ऋतजात ऋतावरि मधु मे मधुला करः ॥२॥

dve cha me viṁśatiśchame'pavaktāraōṣadhe;
ṛtajātaṛtāvarimadhu me madhulākaraḥ.

[Meaning] O herb used in performing yajña and full of juicy potentialities make us regain health if we are attacked by 2 diseases and (2×10^1) twenty.

तिस्रश्च मे त्रिंशच्च मेऽपवक्तार ओषधे ।

ऋतजात ऋतावरि मधु मे मधुला करः ॥३॥

tisraścha me triṁśachchame'pavaktāraōṣadhe;
ṛtajātaṛtāvarimadhu me madhulākaraḥ.

[Meaning] O herb used in performing yajña and full of juicy potentialities make us regain health if we are attacked by 3 diseases and (3×10^1) thirty.

चतस्रश्च मे चत्वारिंशच्च मेऽपवक्तार ओषधे ।

ऋतजात ऋतावरि मधु मे मधुला करः ॥४॥

chatasraścha me
chatvāriṁśachchame'pavaktāraōṣadhe;
ṛtajātaṛtāvarimadhu me madhulākaraḥ.

[Meaning] O herb used in performing yajña and full of juicy potentialities make us regain health if we are attacked by 4 diseases and (4×10^1) forty.

पञ्च च मे पञ्चाशच्च मेऽपवक्तार ओषधे ।
ऋतजात ऋतावरि मधु मे मधुला करः ॥५॥

pañcha cha me pañchāśachchame'pavaktāraōṣadhe;
ṛtajātaṛtāvarimadhu me madhulākaraḥ.

[Meaning] O herb used in performing yajña and full of juicy potentialities make us regain health if we are attacked by 5 diseases and (5×10^1) fifty.

षड्च मे षष्टिश्च मेऽपवक्तार ओषधे ।
ऋतजात ऋतावरि मधु मे मधुला करः ॥६॥

ṣaṭcha me ṣaṣṭiśchame'pavaktāraōṣadhe;
ṛtajātaṛtāvarimadhu me madhulākaraḥ.

[Meaning] O herb used in performing yajña and full of juicy potentialities make us regain health if we are attacked by 6 diseases and (6×10^1) sixty.

सप्त च मे सप्ततिश्च मेऽपवक्तार ओषधे ।
ऋतजात ऋतावरि मधु मे मधुला करः ॥७॥

sapta cha me saptatiśchame'pavaktāraōṣadhe;
ṛtajātaṛtāvarimadhu me madhulākaraḥ.

[Meaning] O herb used in performing yajña and full of juicy potentialities make us regain health if we are attacked by 7 diseases and (7×10^1) seventy.

अष्ट च मेऽशीतिश्च मेऽपवक्तार ओषधे ।
ऋतजात ऋतावरि मधु मे मधुला करः ॥८॥

aṣṭa cha me'śītiśchame'pavaktāraōṣadhe;
ṛtajātaṛtāvarimadhu me madhulākaraḥ.

[Meaning] O herb used in performing yajña and full of juicy potentialities make us regain health if we are attacked by 8 diseases and (8×10^1) eighty.

नव च मे नवतिश्च मेऽपवक्तार ओषधे ।
ऋतजात ऋतावरि मधु मे मधुला करः ॥९॥

nava cha me navatiśchame'pavaktāraōṣadhe;
ṛtajātaṛtāvarimadhu me madhulākaraḥ.

[Meaning] O herb used in performing yajña and full of juicy potentialities make us regain health if we are attacked by 9 diseases and (9×10^1) ninety.

दश च मे शतं च मेऽपवक्तार ओषधे ।
ऋतजात ऋतावरि मधु मे मधुला करः ॥१०॥

daśa cha me śataṁ cha me'pavaktāraōṣadhe;
ṛtajātaṛtāvarimadhu me madhulākaraḥ.

[Meaning] O herb used in performing yajña and full of juicy potentialities make us regain health if we are attacked by 10 diseases and (10×10^1) hundred.

शतं च मे सहस्रं चापवक्तार ओषधे ।
ऋतजात ऋतावरि मधु मे मधुला करः ॥११॥

śataṁ cha me sahasraṁchāpavaktāraōṣadhe;
ṛtajātaṛtāvarimadhu me madhulākaraḥ.

[Meaning] O herb used in performing yajña and full of juicy potentialities make us regain health if we are attacked by 100 diseases and (100×10^1) thousand.

In the above-cited verses, it is revealed that when zero is placed to the right of the numbers their power is added increased by 10 times. The above verse explicitly reveals:

1×10^1 10

2×10^1 20

3×10^1	30
4×10^1	40
5×10^1	50
6×10^1	60
7×10^1	70
8×10^1	80
9×10^1	90
10×10^1	100
100×10^1	1000

The above mantras also indicate:

That when zero is placed to the right of one (1), the power of one (1) is increased by 10 times turning one into ten, similarly zero makes two as twenty; three as 30; four as forty; five as fifty; six as sixty; seven as 70; 8 as 80; 9 as 90; 10 as 100 and 100 as 1000.

Here in the above-cited verses, we find not only the numbers of two digits but also the numbers of three and four digits. By this description, it is quite clear that the rule adopted in making various numbers from 10 to 1000 does not stop on 1000 but also may be applied further in making the numbers of five, six digits, and further on.

Thus from the aforementioned discussion on powers of ten, place value notions, and the base 10 decimal system, it is crystal clear that the invention of zero is a singular contribution of India to the modern world of Science and technology.

Ṛgveda: Zero a tool to increase the value or magnitude

Vedic seers found in zero a tool to increase the value or magnitude of numbers. When a zero is placed just in the right position of the units 1 to 9 increases their values ten times and the same process makes hundred and thousand from ten and hundred respectively, is described in the following verses of the Ṛgveda (10.51.3).

ऐच्छाम त्वा बहुधा जातवेदः प्रविष्टमग्ने अप्स्वोधिषु।
तं त्वा यमो अचिकेच्चित्रभानो दशान्तरुष्यादतिरोचमानम्।।

aichchhāmatvābahudhājātavedaḥpravisṭamagneapvotid hiṣu;
taṁtvāyamoachikechchitrabhānodaśāntaruṣyādatiroch amānam.

[Meaning] We desire you, O Agni Jātavedas (solar energy which is known by its appearance alone), which has variously permeated into waters and plants, etc. Yama (your concealed nature) is recognized or perceived by your illumination, just as zero remains (antaruśyam) concealed into numbers and known when it increases the power of a digit by 10 times from place to place in a very interesting way.

In the above mantra, in the following hemistich,

दशान्तरुष्यादतिरोचमानम्।

daśāntaruṣyādatirochamānam

the significance of zero is described. Accordingly, due to zero, the power of numbers is interestingly increased by tens from place to place.

Sāyaṇa in his commentary on the above Mantra points out the factthat zero remains concealed and increases the magnitude of numbers by 10 times.

अन्तरुष्यं गूढमावासस्थानम्। तच्च च स्थानं दशसंख्योपेतम्।
antaruṣyaṁgūḍhamāvāsasthānam;
tachcha cha sthānaṁdaśasaṁkhyopetam.

[Meaning] The zero remains hidden in the numbers. It is visible when it increases the value of numbers by 10 times from place to place.

Confirming the above fact, the *Viṣṇu-Purāṇa* (6.3.4) observes in the same manner:

स्थानात्स्थानं दशगुणमेकस्माद्गण्यते द्विज।
ततोऽष्टादशमे परार्धमभिधीयते।।

sthānātthāsnaṁdaśaguṇamekasmādgaṇyatedvija;
tato'ṣṭādaśameparārdhamabhidhīyate.

[Meaning] O Dvija, from the place to the next in succession, the places are multiples of ten. The eighteenth one of these (places) is called parārdha."

Using a simile, Vyāsa in his commentary on Patañjali's Yoga-sūtra (3.13) says that the same rekhā (digit) is termed one in the units place, ten in the tens place, and hundred in the hundreds place. The actual reference is cited below.

यथैका रेखा शतस्थाने शतं दशस्थाने दशैका चैकस्थाने ।

yathaikārekhāśatasthāneśataṁdaśasthānedaśaikāchaikas thāne.

[Meaning] Just as arekhā (a digit) connotes a hundred in the 'hundreds' place, ten in the 'tens' place and one in the 'units' place.

We also come across a very interesting śloka in the *Samayochita Padma-ratna-mālikā* (śloka No. 48), a collection of relevant ślokas published in Mumbai in

1957, where it states that when zero is placed to the right of numbers their power is increased by ten times. The verse goes like this.

अंकेषु शून्यविन्यासाद्, वृद्धिः स्यात् तु दशाधिका ।
तस्माद् ज्ञेया विशेषेण, अंकानां वामतो गतिः ॥

aṁkeṣuśūnyavinyāsād, vṛddhiḥsyāttudaśādhikā;
tasmādjñeyāviśeṣeṇa, aṁkānāṁvāmatogatiḥ.

[Meaning] When zero is placed to the right of numbers their power is increased by ten times. That is why the numerals are placed from left to right.

Nature and Philosophy of Zero

In the Vedic literature, the words used to express zero also points out its nature and philosophy as viewed by the Vedic seers.

1. Zero is like space and Brahman

For instance, in the *Yajurveda* (40.17), the nature of zero has been defined by the term 'kha' associating it with Brahman.

ओं खं ब्रह्म।

Oṁkhaṁ brahma

[Meaning] Brahman pervades the whole universe, just as zero permeates all the numbers.

Brahman is all prevalent, ananta (infinite), aparimita (having no boundaries or limit), aprimeya (immeasurable), asaṁkhyeya (uncountable), and pūrṇa (full/absolute).

1. Just as Brahman pervades everything but remains invisible, similarly zero also permeates all numbers but it remains invisible. To understand the nature of zero or Brahman, we shall have to understand the Paṇini'ssūtra (1.1.60).

अदर्शनं लोपः।

adarśanaṁlopaḥ

[Meaning] In the grammatical process, lopa means non-visibility, but non-visibility does not mean absence.

Similarly, if zero is not visible in numbers written from left to right, like 1, 2, 3, 4, 5, 6, 7, 8, or 9 that does not mean that zero is missing. Zero is present in all numbers but not visible, like Brahman. The Brahman is not visible, but it never means that Brahman is missing from the creation.

The civilizations that could not identify the invisible Brahman, also could not invent zero.

2. Brahman is infinite, immeasurable, uncountable, and absolute, similarly zero in the Vedic parlance is not empty but infinite, immeasurable, uncountable, and absolute. The most popularly used word these days for zero is 'śūny' which has sprung from the Vedic word 'śūna' meaning 'fully blown' or 'fully fledged'. Just as the space is not empty but full of all ingredients of creation, similarly zero is not empty.

3. Brahman is pūrṇa (absolute), similarly zero is absolute. The absolute value of 0 is 0.

4. Just as Brahman alone survives after the whole world is eliminated, similarly, when all numerals are eliminated, zero alone survives.

The famous Upaniṣadic quote reads as follows:

ॐ पूर्णमदः पूर्णमिदं पूर्णात्पूर्णमुदच्यते ।
पूर्णस्य पूर्णमादाय पूर्णमेवावशिष्यते ॥

pūrṇamadaḥpūrṇamidaṁpūrṇātpūrṇamudachyate.
pūrṇasyapūrṇamādāyapūrṇamevāvaśiṣyate.

[Meaning] Brahman is pūrṇa (fullness/absolute), this space is pūrṇa (fullness/absolute), If pūrṇa (fullness) of space is taken out of pūrṇa (fullness of Brahman), pūrṇa (the fullness) does remain.

Inversely this statement also applies to zero. Mathematically it points out $0-0=0$ or $\infty - \infty = \infty$

5. Just as Brahman is invisble, zero also represents invisible. In Bakhśāli Mathematics (200 AD) the invisible quantity is represented by symbol '0' which is called 'śūnya' (zero). This is obvious from the folio 22 verso

which reads as under:

द्विगुणं द्वितीयस्य प्रथमा----तीय। प्रथमा चतुर्गुणं चैव चतुर्थे चैव दत्तवान् च शतमेकं द्वयानुगम। वदस्व प्रथमे दत्तं किं प्रमाणम्-

dviguṇaṁdvitīyasyaprathamā----tīya. prathamā chaturguṇaṁchaivachaturthechaivadattavān cha śatam ekaṁdvayānugama. vadasvaprathamedattaṁkiṁ pramāṇam-

स्य···	0 1	2 1	3 1	4 1	दृश्य 200 (invisible 200) 1	शुन्यमेकयुतं कृत्वा (adding unity to invisible/ zero)
	1	2	3	4	···क्षेप युक्त्या फलम् (having added 1 to invisible)	
	20 आ 20	40 उ 20	60	80	एवं 200 Thus the share is 200	
			प 4			
	1	1	1			

The above problem is of this type:

"A certain amount was given to the first invisible person, twice of invisible to the second, thrice to the third, and four times to the fourth. State the amount given to the invisible and the shares of others, if the total visible amount (given amount) was 200."

In the above problem, shares are represented by 0, 1, 2, 3, and 4 (Here zero stands as a symbol for invisible).

Now having added 1 to the invisible (0) the sum is 1+2+3+4= 10, which shows the proper share of the first is 200/10 or 20 and the series is 20+40+60+80= 200.

Here Hörnle (1983:90; 1988:30) and Kaye (1912:357) consider the 'śūnya' as the symbol of 'unknown'. But here the invisible person represents the invisible Brahman which is represented by śūnya and nothing else. The same śūnya has also been used for the zero for the decimal arithmetical notations. This is, indeed, its true significance. Its mention in connection with an algebraic equation, in a sense other than for arithmetical notation, is simply to indicate that the quantity which is invisible so not known. The following examples clarify this issue further.

Folio 13, verso quotes मूलं न ज्ञायते (*mūlaṁnajñāyate*)

15 verso quotes प्रथमं न जानामि *(prathamaṁnajānāmi)*

24 verso quotes पदं न ज्ञायते *(padaṁnajñāyate)*

In each case cited above, unfamiliarity with the invisible elements has been pointed out and the same invisibility has been indicated by śūnya.

Thus on the basis of aforesaid citations, it can safely be maintained that zero has its philosophical origin in India. Its characteristics are closely related to those of Brahman.

1. Like a yogī when keep meditating on infinite Brahman, merges with the infinite power of Brahman, in a similar manner you keep on suffixing zero to a number to increase its power to infinity (∞).

2. Brahman permeates all living beings and non-living objects, but His presence does not affect the individuality of a living being or non-living object, similarly zero when added or subtracted to a number it makes no

difference in the individual power of a number.

3. When a yogī through samādhi merges with Brahman, he loses his individual identity or say as an individual he is reduced to zero and becomes one with Brahman, similarly, when zero is multiplied to a number it reduces its power to zero.

4. The realization of Brahman increases the power of the seeker to infinity, similarly when zero divides a number, it increases its power to infinity (∞).

2. Zero represents a transitional stage

Zero is sandwiched between +ve and -ve powers or numbers. It is a transitional stage, when you switch over from +ve to -ve you shall have to pass through it.

This nature of zero has been cited in the following verse of the *YogavāsiṣṭhaMahārāmāyaṇa* (NirvāṇaPrakaraṇa, Uttarārdha, 127.20).

पर्यन्ते तस्य नभसः स्थितं ब्रह्माण्डखर्परम् ।
एकमूर्ध्वे परमधो गगनं मध्यमेतयोः ॥

paryantetasyanabhasaḥsthitaṃbrahmāṇḍakharparam,
ekamūrdhveparamadhogaganaṃmadhyametayoḥ.

[Meaning] At the end of this sphere, there is the great circle of the universe; having one half of it stretching above and one below, and containing the space in the midst of them like that of zero between +ve and -ve numbers.

This śloka gives the origin of zero and +ve and –ve numbers. +ve numbers are heavenly numbers and –ve numbers are infernal regions. Space representing zero is sandwiched between them.

3. Zero is not a countable number

The following verse of the philosophical treatise the *YogavāsiṣṭhaMahārāmāyaṇa* (NirvāṇaPrakaraṇ a, Uttarārdha, 127.27), speaks of another characteristic of zero giving a simile of Brahman. According to it, zero inherits all countable numbers, yet it cannot be called a countable number.

नभसोऽप्यधिकं शून्यं न च शून्यं चिदात्मकम् ।
सूर्यकान्ते यथा वह्निर्यथा क्षीरे घृतं तथा ॥ 27 ॥

nabhaso'pyadhikaṃśūnyaṃnacaśūnyaṃcidātmakam,
sūryakānteyathāvahniryathākṣīreghṛtaṃtathā.

[Meaning] The Chaitanya or Brahman is more rarefied/subtle than zero or space, and yet zero or space cannot be called Brahman. This is like a sun-stone inheriting in it the fire but fire cannot be a sunstone, and the milk inheriting butter in it, but butter cannot be milk; similarly, God inherits space in it, but space cannot be God and zero inherits all countable numbers in it, but countable numbers cannot be zero.

4. Zero is contained in zero

At another place, the *Yogavāsiṣṭha Mahārāmāyaṇa* (Nirvāṇa Prakaraṇa, Nirvāṇa Prakaraṇa, Pūrvārdh, 2.11), speaks of yet another characteristics of zero. Accordingly, zero is contained in zero. There is no other container of zero, except zero itself.

शून्यं शून्ये समुच्छूनं ब्रह्म ब्रह्मणि बृंहितम् ।
सत्यं विजृम्भते सत्ये पूर्णे पूर्णमिव स्थितम् ॥
śūnyaṃśūnyesamucchūnaṃ brahma
brahmaṇibṛṃhitam,
satyaṃvijṛmbhatesatyepūrṇepūrṇamivasthitam.

Just as the śūnya (zero) is contained in zero; the vastness of Brahman is contained in the immensity of Brahman, and as truth can exist in truth; so infinity (∞) is contained in the infinity.

5. Addition of infinite numbers of zeros makes one zero

According to the *Yogavāsiṣṭha Mahārāmāyaṇa* (Nirvāṇa Prakaraṇa, Nirvāṇa Prakaraṇa, Pūrvārdh, 3.14), infinite numbers of zeros make only one zero.

यथा नानाप्यनानैव खं खे खानीति वाग्गणः ।
सार्थकोऽप्यतिशून्यात्मा तथात्मजगतोः क्रमः ॥

yathānānāpyanānaivakhaṃkhekhānītivāggaṇaḥ,
sārthako'pyatiśūnyātmā tathātmajagatoḥ kramaḥ.

[Meaning] Just as the one, two, or infinite numbers of zeros make one zero, although this classification of one, two, or more zeros is meaningful and justified in statements. Similarly, the infinite numbers of spaces make one space and the infinite number of ātmans and worlds make one Brahman.

The Binary Number System and Zero

The binary number system is a system in which numbers are represented as linear combinations of the powers of 2.

This is a positional numeral system employing only two kinds of binary digits, viz. 0, 1. The importance of this system lies in the convenience of representing decimal numbers using a two-state system in computer technology. The system was in vogue during the Vedic period also.

Vedic system

अस्ति (*asti*)	नास्ति (*nāsti*)
आग्नेय (*āganeya*)	सोमीय (*somīya*)
सत्य (*satya*)	मिथ्या (*mithyā*)
ज्य◌ोतिष (jyotiṣa)	आप◌ः (*āpaḥ*)
पुरुष (*puruṣa*)	प्रकृति (*prakṛti*)
लघु (*laghu*)	गुरू (*gurū*)

Modern system

On	off
open	closed
charged	non-charged in electronic circuitry

The Discovery of binary numbers is attributed to Gottfried Leibniz (1646-1716) at the end of the 17th century. He is said to have come up with the idea when he interpreted Chinese hexagram depictions of Fu Hsi in I-Ching (the book of changes) in terms of binary code. But In India Āchārya Piṅgal (28 century B.C. as per Indian tradition/2nd century B.C. as per western historians), the author of the *Chandaḥ Śāstra* and a mathematician and meterist applied binary code as many 2800 years before Christ while suggesting the *prastara* of *chandas.*

Piṅgala gives the following rule:

द्विरर्द्धे -8.28

Place two when halved

रूपे शून्यम् -8.29

When rūpa (unity or one) is subtracted, place zero

द्वि शून्ये -8.30
Multiply by two when zero

तावद् अर्द्धे तद् गुणितम् -8.31
Square when halved

The Binary Code in Prastar of Gāyatrī Chanda	**No**	**No to be placed**
No of syllables	6	
Halve the no.	6/2=3	2
Since 3 cannot be halved, subtract 1	3-1=2	0
Halve the number	2/2=1	2
Since 1 cannot be halved, subtract 1	1-1=0	0

Set (2,0,2,0). Let us start from last no. 0 and apply the Piṅgala rule

2	0	2	0

$(2^3)^2= 2^6$	$2x2^2 =2^3$	2^2	2

26= 2 x 2 x 2 x 2 x 2 x 2= 64

Similarly, we can have binary code for other chhandas

Uṣṇika = 7 = 0,2,0,2,0= 2^7= 128

Anuṣṭup= 8= 2^8= 256

Bṛhati= 9= 2^9= 512

Paṅkti=10=210= 1028

Triṣṭup= 11= 2^{11}= 2056

ati= 12 = 2^{12}= 4196

Various Denominations of Zero

Zero has been represented by various denominations starting from its Vedic denomination 'kha' to 'śūnya'. It will be highlighted by the following citations:

'Kha' 'Śūnya' and 'Ambara'

We have seen while applying binary code ĀchāryaPiṅgala (28th century traditional/200 BC modern) used for the first time the word 'śūnya' for zero. Afterward, the word śūnya'in addition to 'kha' and its synonyms was also applied to express zero in India.

In the *Pañcha-siddhāntikā* (649 BC traditional estimate & 500 AD modern estimate), Varāhamihira addresses zero by denominations like 'kha', 'śūnya' and 'ambara'. For instance in 3.2 and 3.17, 'kha' is used to describe zero. Examples are:

एकादशाऽष्टषट् रूपोना सप्ततिः 'ख'युक्ता च। 3.2

ēkādaśā'ṣṭaṣaṭrūponāsaptatiḥ 'kha'yuktā cha। 3.2

[Meaning] The numbers of minutes to be deducted from the Sun's mean longitude) are eleven (11), eight sixes (48), seventy minus one सप्ततिः रूपोना (69), and that plus zero खयुक्ता (70).

[यम]- शिखि-गुणा-ऽग्नि-यम-शशि-वियुता सैका सरूप रूपै-का
खै-कवियुता च भानोः षष्टिर्भुक्तिः क्रमादेवम्।। 3.17

[*yama]- śikhi-guṇā-'gni-yama-śaśi-*
viyutāsaikāsarūparūpai-kā;
khai-kaviyutā cha bhānoḥṣaṣṭhirbhuktiḥkramādevam.

[Meaning] The daily motion of the sun in minutes during each of 12 months amounts to sixty minus three (57), three (57), three (57), three (57), two (58), one (59), plus one (61), one (61), one (61), one (61), minus kha (60), one (59) in turn.

Note: In the above śloka it has also been pointed out that when zero is subtracted from a number, its value does not change.

In 4.7 and 6.12, 'śūnya' is used to denote zero. Examples are:

सैकाऽजे पंचाशत् पंचाष्टकं पंचवर्गवेदाश्च ।
त्रिंशच्चतुर्भिरधिका षद्वंचशच्छराः शून्यम् ।। 4.7

saikā'jepaṁchāśatpaṁchāṣṭakaṁpañchavargavedāścha; triṁśachchaturbhiradhikāṣaṭpaṁchaśachchharāḥśūnya m.

[Meaning] In the first sign, the second parts are successively fifty plus one (51), five eights (40), five squared (25), four, thirty increased by four (34), fifty plus six (56) five (5) and śūvya (0).

In 4.8, the denomination of 'ambara' is used to denote zero. For example,

षट् त्रयोदशै (कोना) विंशति स्त्रयष्टकान्यतस्त्रिंशत् ।
युक्ताम्बर-पंचनवा (तिज) गतिभिर्लिप्तिका वृषभे ।। 4.8

ṣaṭtrayodaśai (konā) viṁśatistrayaṣṭakānyatastriṁśat; yuktāmbara-paṁchanavā (tija) gatibhirliptikāvṛṣabhe.

[Meaning] Of the sines in the second sign, the minutes' parts taking the increments in the current sign alone, are successively six (6), thirteen (13), twenty minus one (19), three into eights (24) thirty plus ambara (zero) (30), thirty plus five (35), and thirty plus nine (39).

Note: The above example also indirectly informs that when zero is added to a number, its value remains unchanged.

Thus we see in one single book three denominations were used to express zero numerals.

All the above instances occur in those sections of the Pañcha-siddhāntikā which deal with the teachings of Pauliśa. It seems, therefore, that such expressions are quotations from the Pauliśa-siddhānta. Pauliśa-siddhāntahas now become extinct. Varahamihira (649 BC as per Indian traditional Chronology & 500 AD by modern scholars) in his *Pañcha-siddhāntik*ā mentions five (*pañcha*) ancient Siddhantas arranged in order of chronology. They were the Paitāmaha, Vasiṣṭha, Romaka, Pauliśa, and Sūrya. Presently none of Siddhāntas has survived except Sūrya-siddhānta. Sūrya-siddhānta's ancient version has also become extinct, only its modern version is available. From the aforementioned, we can understand the antiquity of Pauliśa-siddhānta (27th Mahāyuga traditional/400 AD modern) which was ancient than the ancient Sūrya-siddhānta. The text of Sūrya-siddhānta says that the Sūrya-siddhānta was revealed to Maya at the end of the Satyayuga of the present 28th Mahāyuga. As such we can safely say that Pauliśa-siddhānta was prevalent in this country in the hoary past. Now it has survived in the references quoted by ancient and medieval astronomers like Bhaṭotpala (305 A.D. traditional). It is also well known that 'word numerals' were employed by Pauliśa, so it can be safely concluded that he was conversant with the concept of the zero as a numeral.

The writings of Jinabhadra Gaṇi, a contemporary of Varāhamihira, offer conclusive evidence of the use of zero as a distinct numerical symbol. While mentioning large numbers containing several zeros, he often enumerates, obviously for the sake of abridgment, the number of zeros contained. For instance: 224,400,000,000 is mentioned as "twenty-two forty-four, eight zeros;" (*Bṛhat-kṣetra-samāsa*, 1. 69)

द्वि विंशति च चतुश्चत्वारिंशति च अष्ट शून्यानि

dvi viṁsati ca chatuśchatvāriṁśati ca aṣṭaśūnyāni

[Meaning] Twenty-two and forty-four and eight zeros.

Similarly, 3,200,400,000,000 is mentioned as "thirty-two, two zeros, four, eight zeros. (*Bṛhat-kṣetra-samāsa*, 1. 71). There are several instances of this kind in his work (1.90, 97, 102, 108, 113, 119, etc.)

At another place in the *Bṛhat-kṣetra-samāsa* (1.83) zero is mentioned in describing

$$241960\ \frac{407150}{483920} = 241960\ \frac{40715}{48392}$$

as two hundred thousand, forty-one thousand, nine hundredand sixty; removing (*apavartana*) the zeros, the numerator is 'four-zero-seven-one-five', and the denominator 'four-eight-three-nine-two'.

Here the term *apavartana* is used in the sense of the reduction of a fraction to its lowest terms by removing the common factors from the numerator and the denominator. Hence the zero of JinabhadraGaṇi is a specific numerical symbol used in the arithmetical calculation (Datta, 1962:79).

Another contemporary mathematician, Bhaskara 1 in his famous text *Mahābhāskarīyam* (1.36) refers to the word 'vyoma', a synonym of 'kha' and 'śūnya' and 'bindu' (small circle/raindrop) to represent zero.

व्योमशून्यनेत्रभाजिते फलं राशयोऽष्टभाजितेऽथ लिप्तिकाः ।
बिन्दुषड्गुते विलिप्तिका विदुः सर्वमेव योज्य गण्यते बुधः ।।

vyomaśūnyanetrabhājitephalaṁrāśayo'ṣṭabhājite'thaliptikāḥ;

binduṣaḍḍhṛteviliptikāviduḥsarvamevayojyagaṇyatebu dhaḥ.

[Meaning] Divide grahatanu by 200 (vyoma-śūny-netra); the result is in terms of rāśīs. The divide by 8, we get minutes (liptikā); then divide by 60 (bindu-ṣad), we get seconds. Adding all these (including 4 times the Sun's mean longitude as prescribed in the previous śloka) we get the mean longitude (śighrochcha) of Mercury.

In the above śloka, vyoma and śūnya both have been used to represent zero (0) making the number 200. Similarly, zero of sixty (60) has been expressed by the word 'bindu'. So we can say that 'bindu' was an additional denomination for representing the numeral zero (0).

We come across a passage from the Vāsavadattā of Subandhu where the stars have been compared with 'śūnya-bindus' (zeros). Thus, Subandhu in his book 'Vāsvadattā' uses the word 'bindu' (literally meaning small circle/raindrop, etc.) as a symbol to represent zero. The passage goes like this:

शशिकठिनोखण्डेन तमोमशीश्यामेऽजिन इव नभसि संसारस्यातिशून्यत्वाच्छून्यबिन्दव इव....

śaśikaṭhinokhaṇḍenatamomaśīśyāme'jinaivanabhasisa ṁsārasyātiśūnyatvāchchhūnyabindavaiva....

[Meaning] And at the time when the Moon rises with its blackness of night, bowing down under the guise of closing blue lotuses, the stars appear in the sky like śūnya-bindus (zeros)...

Subandhu was the minister of King Bindusāra (15th century B.C. as per Indian tradition/280 BC as per European scholars) of Pāṭaliputra. The famous Patañjali (1200 B.C. as per Indian tradition/150 B.C. as per

European scholars)makes a mention of Subandhu'sVāsvadattā.

In his commentary on the gloss of Umāsvāti on Tattvaārthādhigama-sūtra (3.11), SiddhsenaGaṇiuses the word śūnya for zero while performing the square root of 3,53,44,00,00,00,00 as 1880000 by removing the four śūnyas' (zeros) from eight and doing the square root of remaining portion 35344 into 188. According to Jain tradition, Umāsvāti lived around 200 BC. The time of Siddhsena is considered to be around the 4th century AD.

Bhāskara II and so many other scholars also extensively use the word 'kha' as the denomination of zero. Bhāskara II uses the word kha-guṇa for the product of a number and zero and 'kh-hara' for the division of a number by zero

Thus we see that 'kha' and synonyms of 'kha' like 'ambara', 'vyoma', 'ākāśa', etc. were used as the denominations of zero right from the Vedic age till the medieval period of Indian history, but the use of 'śūnya' started by ĀchāryaPiṅgalain the post-Vedic period and became more popular now-a-days replacing all other denominations.

Palaeographic evidence of the use of Zero Symbol

The Bakhśālī Manuscript (200 AD) was found in 1881 at a place called Bakhśālī, about eighty km. North-East of Peshawar (now in Pakistan), by a farmer who dug it up while cultivating the land. The manuscript is preserved in the Bodleian Library at Oxford University. The untitled manuscript, written on birch bark, is composed in the Gāthā language (a modified form of prākṛta) and the scripts used are recognized as an earlier type of Śāradā, once used in the Kashmir region of India. In the seventy fragmentary leaves of the document, which is now extant, one finds numerous intricate calculations involving decimals, and place-value notation of numerals, including a solid dot and a small circle for śūnya (zero). The work seems to be an exercise compiled by a scribe. The manuscript records the word śūnya to represent invisibly (zero) in calculations. For instance, on folio 56 verso, we have:

"880 x गुणितं जातं (multiplied by) 964 = 848320

84 x गुणितं जातं (multiplied by) 168 = 14112

चत्वारिंश पृथक् स्थानं वर्ग (the square of 40 different places is) 1600

एष उपरापात्यशेषं (On subtracting this from the number above (numerator), the remainder is

848320-1600= $\frac{846720}{14112}$ = 60

वर्त्यजातं(on the removal of the common factor it becomes 60

For an explicit reference to zero and an operation with it, folio 22, verso has already been cited above.

On the other hand, there are positive palaeographical shreds of evidence of the use of zero as a decimal digit depicted as a small circlein the two stone inscriptions of Gwalior from King Bhojadeva (336-392 A.D.) discovered in the 19th century in a temple of Lord Viṣṇu in Gwalior fort, and KingMahipāla (Ojha. 1918: 127 &Lipi-patra 75).

We also find it used in Kalchuri grant dated 695 AD (Ojha, 1918: 115).

Origin of Zero Symbol

Throughout Vedic and post-Vedic Sanskrit literature, zero was expressed by the word 'kha' so its symbol was also developed based upon the philosophy behind the word 'kha'. 'Kha' also means space and space have been characterized as 'Khagola' meaning 'round or egg-shaped universe'. Its synonym is Brahmāṇḍa, which also literally means 'Egg-shaped Universe'. Since 'kha' or the universal space as well as chidākāśa (space of Brahman) from which the universal space manifested, both are 'gola' 'round or egg shaped', so zero was also represented by a 'small circle' or 'round' or 'egg shaped' figure (0). This figure is applied right from the Vedic period to date. In ancient India, zero was also denoted by the word 'Pūrṇa' which represents Pūrṇa Chandra 'Full Moon'. So zero was also shaped like 'Full Moon'. In Gopatha Brāhmaṇa (2.2.5), we come across an example where kha (zero) is described as a 'chhidra' (hole). The reference is cited below:

मख इत्येतद्यज्ञनामधेयम्। छिद्र प्रतिषेधसामर्थ्यात् छिद्रं खं इत्युक्तम्। तस्य मा इति प्रतिषेधो मा यज्ञं छिद्रं करिष्यतीति।

makhaityetadyajñanāmadheyam. chhidrapratiṣedha-sāmarthyāt chhidraṁ khaṁ ityuktam. tasyamāitipratiṣedhomāyajñaṁchhidraṁkariṣyatīti. chhidramkhamityutkam

[Meaning] Yajña is called makha (ma+kha). 'Ma' represents 'Mā' which means 'prohibition' and 'kha' is used to represent zero (emptiness). Zero has the shape of chhidra (hole). So 'ma-kha' means prohibition of chhidra (hole or emptiness) gap-filler. As such yajña fills up zero orchhidra (emptiness).

The Amarkośa also describes zero as 'randhra (hole).

In addition to the above examples, we find some alterations in its shape or figure. As pointed out above there are positive palaeographical shreds of evidence that zero was represented in India by a small circle in the two inscriptions of Bhojadeva (336-392 A.D.), and Mahipāla (Ojha. 1918: 127 &Lipi-patra 75). However, in Bakhśāli Manuscript, we find zero represented by bindus or solid dots (.). Bindu means 'rain-drop'. Raindrop is a circular shaped figure. So, zero was also represented by circular-shaped figures (0) or dots (.). Solid dots were easily scribed on birchbark instead of a circular-shaped figure (0), so the scribe would have used bold dots for the sake of convenience. However, Mukherjee (1991) identifies one particular solid dot in the manuscript as a small circle. So, using a solid dot for the sake of convenience in writing on birchbark, never means that the scribe was not aware of the circular-shaped figure of zero (0).

As already pointed out above the *Vāsavadattā* of Subandhu in a metaphor points out bindu (solid dot/drop) as a representative figure for 'śūnya' (zero) while comparing the stars with 'śūnya-bindus' (bindus as figures representing zero). Thus, Subandhu in his book 'Vāsvadattā' uses the word 'bindu' (literally meaning dot, raindrop, etc.) as a symbol to represent zero. In Naiṣada-charita (1.21) also, ŚriHarṣa (88-92 A.D./Modern 606 A.D.) describes zero as śūnyabindu' as under:

किमस्य लोम्नां कपटेन कोटिभिर्विधिर्न लेखाभिरजीगणद्गुणान् ।

न रोमकूपौघमिषाज्जगत्कृता कृताश्च किं दूषणशून्यबिन्दवः ॥ २१ ॥

kimasyalomnāṃkapaṭenakoṭibhirvidhirnalekhābhirajīg aṇadguṇān;
naromakūpaughamiṣājjagatkṛtākṛtāścakiṃdūṣaṇaśūnya bindavaḥ.

[Meaning] Did not the Creator reckon his merits with crores of lines, the hairs of his body? Did not the maker of the world put the pores of his skin for a small circle of zeros to indicate the absence of defects?

Here it may be pointed out that the above examples may not be taken so seriously to decide the form of the zero symbols. Bindu has also been used in the sense of raindrop, spot as in *bindu-mṛga* (spotted deer) and small circle.

Journey of Zero from the East to the West

The science of Mathematics too in India goes back to Vedic times as in the case of other sciences. Algebra, Arithmetic including geometry had their origin in India. India is the inventor of many mathematical ideas. These ideas developed during the Vedic period and percolated with the passage of time to the present age. A lot of basic mathematical ideas travelled from India to other parts of the world. There are many examples that shed ample good light on the Indian influence on Arabic, Roman and Greek mathematics.

The sexagesimal place value notation of Hellenistic astronomy was a modification of Indian decimal and place value notation.

The trapezium problems in Heron can be traced to Āryabhaṭa. Brahmagupta's treatment of rational solutions of the right-angled triangle influenced Ptolemy's cyclic surds, and Euclid's quadrilaterals and influenced Diophantus' indeterminate equations of the second degree in Greece. Pythagoras' theorem can be traced back to BaudhāyanaŚulvasūtras.

It is a proven fact that there was close contact between the Indian and Greek mathematicians in those old days.

The statement of a Syrian astronomer (John Leyden et. al, 1921: 78-79) of 7th century A.D. is noteworthy. He writes-

> "No word can praise strongly enough the knowledge of the Hindus. If these things are known to the people who think that they alone have mastered the sciences because they speak Greek, they would

perhaps be convinced though a little late in the day that other folk not only Greek but men of different tongues, know something as well as they."

Like all other Indian mathematical inventions 'śūny' (zero) also reached Arab and was transliterated into Arabic in Al-Khwarizmi's (780-850 CE) work '*De NumeroIndorum*'(Hindu Art of Reckoning[1]) as 'sifr', meaning 'empty'. The associated Arabic root √sfr leads to other words of similar meaning, like 'asfara' (meaning 'empty') or safir (meaning 'to be empty'). The slightly changed term 'sifra' was used to denote 'zero' in '*Sefer ha misper*' (meaning, 'Book of Numbers') by Rabbi Ben Ezra (1092-1167). This one, along with many local variants like cifra, cyfra, cyphra, zyphra, etc., was used to denote 'zero'. However, they took a turn by the beginning of the Renaissance and one may find the Spanish word 'cifra', derived directly from Arabic 'sifr' being used with a changed connotation to refer to the rest of the digit numbers. Similar examples prevail in the case of other European languages as well—for instance, 'chiffre' in French, or 'ziffer' in German are still used to refer to digit numbers. However, the Arabic 'sifr' was gradually latinized as 'zephyrus',—'the west wind' and Leonardo of Pisa, better known as Fibonacci, while writing his seminal book, '*Liber Abaci*' in 1202, coined the term 'zephirum'— mere light breeze— almost nothing, to denote 'zero'. Later it got converted to 'zefiro' in the Venetian dialect of the early Renaissance, and modern 'zero' is a mere contraction of that. The first known occurrence of the modern term 'zero' is found in

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[1]"The Hindu Art of Reckoning" written by Abu Jafar Muh Al-Khwarizmi, in the 9thcentury. It was copied in the 13th century, and translated into English by John N. Crossley and Alan S. Henry in 1990 whcih published in *Historia Mathematica* 17 (1990), 103-131.

an Italian book '*De Arithmetica Opusculum*' by Filipps Calandri, published in 1491 in Florence.

On the basis of the above discussion, we can safely say that Indian 'śūnya' reached Arab and became 'sifir' or 'cipher'. From Arab this 'sifir' reached Europe following two routes- one Spanish and another Latin which is illustrated below:

Route No. 1

Sanskrit	**Arabic**	**Spanish**	**French**	**English**
शून्य	सिफ्र	सिप्रा	सिफ्रे	सिफर
Śūnya	Sifir	Cifra	Chiffre	Cypher
		Cyfra		
		Cyphra		
		Zyphra		

Route No. 2

Sanskrit	**Arabic**	**Latin**	**Latin**	**English**
शून्य	सिफ्र	जैफ्रम	ज़ीफ्रो	जीरो
Śūnya	Sifir	Zephirum	Zefiro	Zero

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